The following data were obtained during the calibration of an analytical equipment. *x* is the concentration *y* is the analytical signal.

	x	y	(x-x_avg)^2	y_estimated	(y-y_estimated)^2
	0,109	1482	0,0375	1449	1063
	0,152	1923	0,0227	1981	3319
	0,196	2532	0,0114	2524	61
	0,254	3264	0,0024	3241	543
	0,302	3804	0,0000	3834	882
	0,355	4520	0,0028	4488	996
	0,398	5012	0,0091	5020	59
	0,455	5751	0,0232	5724	738
	0,502	6277	0,0398	6304	754
sum	2,723	34565	0,1487	34565	8414

The parameters of the estimated calibration line:

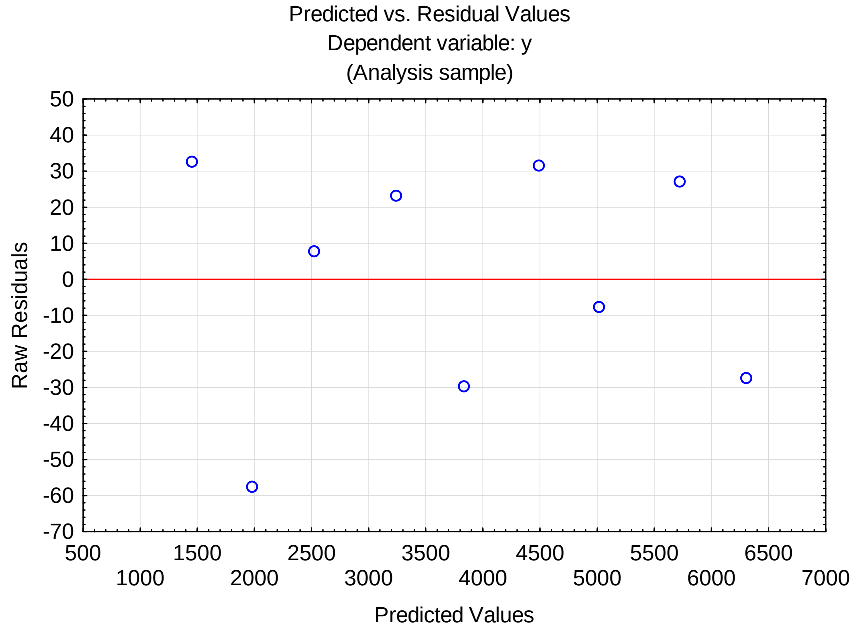
Effect	Parameter Estimates (Spreadsheet14) Sigma-restricted parameterization					
	y Param.	y Std.Err	y t	y p	-95,00% Cnf.Lmt	+95,00% Cnf.Lmt
Intercept	102,82	29,55379	3,4791	0,010279	32,94	172,70
x	12353,88	89,90255	137,4142	0,000000	12141,30	12566,47

a) What is the estimated value of the signal at x=0.32?

b) Would you believe at 0,05 significance level, that the intercept of the true calibration line is 0?

c) The residual plot of the fitted curve is shown above. What do you think, is the fitted linear model adequate?

d) The residual plot of the fitted curve is shown above. What do you think, is the variance of the signal is constant?



e) Give a 95% confidence interval for the expected value of the signal at x=0.32.

f) A new measurement at x=0.32 is 4102. Would you find this surprising?

g) The table below belongs to the estimated calibration line. What does it mean that the Multiple R2 is 0.999815?

Dependent Variable	Test of SS Whole Model vs. SS Residual (Spreadsheet1		
	Multiple R	Multiple R2	Adjusted R2
y	0,999815	0,999629	0,999576

Answers

a) 4056

b) I would not you believe at 0,05 significance level that the intercept of the true calibration line is zero, as the 95% confidence interval of the intercept (32.94, 172.7) does not contain zero.

Or: p=0.010279<0.05 thus the null hypothesis that the true intercept equals to zero is rejected.

c) The points on the figure does not follow any pattern, thus I would believe that the linear model is adequate.

d) The points seems to vary in the

range around zero, thus I would believe that the variance is constant.

e) $s_r^2 = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-2} = \frac{1063 + 3319 + ... + 754}{9-2} = 1202$

$$s^2_{\hat{Y},x=0.32} = s_y^2 \left[\frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \right] = 1202 \left[\frac{1}{9} + \frac{(0.32-0.303)^2}{0.1487} \right] = 135.9$$

$t_{0.025}(7) = 2.365$

$$P\left(4056 - 2.365\sqrt{135.9} < Y < 4056 + 2.365\sqrt{135.9}\right) = 0.95$$
$$P\left(4028.4 < Y < 4083.6\right) = 0.95$$

f) The prediction interval at x=0.32:

$$s^2_{y^*-\hat{Y},x=0.32} = s_y^2\left[1+\frac{1}{n}+\frac{(x-\overline{x})^2}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right] = 1202\left[1+\frac{1}{9}+\frac{(0.32-0.303)^2}{0.1487}\right]=1337.9$$

$$P\left(4056-2.365\sqrt{1337.9} < y^* < 4056+2.365\sqrt{1337.9}\right)=0.95$$

$$P\left(4969.5 < y^* < 4142.5\right)=0.95$$

A new measurement will be in the (4969.5, 4142.5) range with 95% probability. Thus having 4102 as a measured signal at x=0.32 is not surprising. The result can be explained with the fitted calibration line and the random error of the measurement.

g) 99.98% of the variability of the measured values (y) is explained by the fitted line.